# Prediction of Wave Refraction and Shoaling Using Two Numerical Models
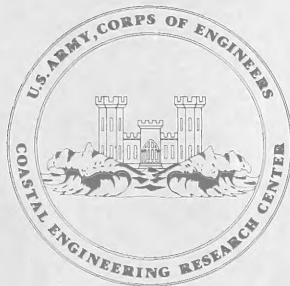
by

Jon M. Hubertz

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20. ABSTRACT *(Continue on reverse side if necessary and identify by block number)*

Two numerical models to predict wave refraction and shoaling in shallow water are described. One model is formulated in terms of wave rays, the other in terms of wave spectra. Output from each model is illustrated and compared to observations made at CERC's Field Research Facility at Duck, North Carolina.

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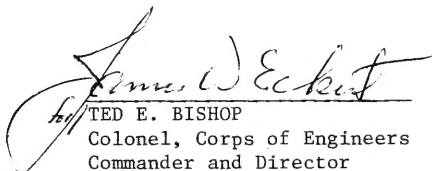
# PREFACE

This report describes two numerical computer models which can be used to estimate the transformation of a wave field from deep to shallow water. The models are based on the same theory as presented in the SPM, but represent an improvement in the technique used to obtain and interpret results. The work was carried out under the waves and coastal flooding program of the U.S. Army Coastal Engineering Research Center (CERC).

The report was prepared by Jon M. Hubertz, under the general supervision of Dr. C.L. Vincent, Chief, Coastal Oceanography Branch, Research Division.

Comments on this publication are invited.

Approved for publication in accordance with Public Law 166, 79th Congress, approved 31 July 1945, as supplemented by Public Law 172, 88th Congress, approved 7 November 1963.

TED E. BISHOP
Colonel, Corps of Engineers
Commander and Director

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FIGURES

# CONVERSION FACTORS, U.S. CUSTOMARY TO METRIC (SI) UNITS OF MEASUREMENT

U.S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

| Multiply | by | To obtain |
|---|---|---|
| inches | 25.4 | millimeters |
| | 2.54 | centimeters |
| square inches | 6.452 | square centimeters |
| cubic inches | 16.39 | cubic centimeters |
| feet | 30.48 | centimeters |
| | 0.3048 | meters |
| square feet | 0.0929 | square meters |
| cubic feet | 0.0283 | cubic meters |
| yards | 0.9144 | meters |
| square yards | 0.836 | square meters |
| cubic yards | 0.7646 | cubic meters |
| miles | 1.6093 | kilometers |
| square miles | 259.0 | hectares |
| knots | 1.852 | kilometers per hour |
| acres | 0.4047 | hectares |
| foot-pounds | 1.3558 | newton meters |
| millibars | $1.0197 \times 10^{-3}$ | kilograms per square centimeter |
| ounces | 28.35 | grams |
| pounds | 453.6 | grams |
| | 0.4536 | kilograms |
| ton, long | 1.0160 | metric tons |
| ton, short | 0.9072 | metric tons |
| degrees (angle) | 0.01745 | radians |
| Fahrenheit degrees | 5/9 | Celsius degrees or Kelvins[1] |

[1] To obtain Celsius (C) temperature readings from Fahrenheit (F) readings, use formula:  $C = (5/9)(F - 32)$.

To obtain Kelvin (K) readings, use formula:  $K = (5/9)(F - 32) + 273.15$.

# PREDICTION OF WAVE REFRACTION AND SHOALING
## USING TWO NUMERICAL MODELS

*by*
*Jon M. Hubertz*

## I. INTRODUCTION

This report discusses two numerical computer models which can be used to estimate the refraction and shoaling of waves from deep to shallow water. The models are based on the methods for estimating refraction due to bathymetry in Section 2.3 of the Shore Protection Manual (SPM) (U.S. Army, Corps of Engineers, Coastal Engineering Research Center, 1977) but have features which make it easier to obtain and interpret the results. Both models estimate wave height and direction in a specified area when provided with an initial wave height and direction on the boundary of the region. One model considers propagation of a monochromatic wave along a ray; the other model propagates frequency components of a wave spectrum.

In the wave ray approach, the movement of a wave front is defined in terms of rays perpendicular to the wave front. Wave heights are calculated along the rays. The model which uses this method is documented by Poole, et al. (1977). In the wave spectra approach, wave height and direction are available at grid points over a specified area as a function of wave frequency. This model is based on the work of Noda, et al. (1974) and described in detail by Wang and Yang (1977). A modification of this program at CERC uses monochromatic wave trains. The monochromatic version is used in this report; however, the spectral version is available.

A major difference between the two models is that in the wave ray approach, values are available only along the rays. In the spectral model (used either spectrally or monochromatically), the wave values are available only at evenly spaced grid points.

## II. APPLICATION OF MODELS

Both models are based on linear wave theory and are limited by assumptions which make that theory valid (see SPM, p. 2-6). Both models assume the conservation of wave energy. In the wave ray model, energy is conserved between two adjacent rays; in the spectral model, energy is conserved within frequency bands. This implies there is no flow of energy between waves of different frequencies. The results of both models are valid only for monochromatic wave trains. The effects of refraction and shoaling on waves of different frequencies can be examined by making multiple runs.

The most important input to either model is the bathymetry specified at grid points over the region of interest. For best results, large variations in depth should not occur over horizontal distances of about one wavelength. It is easier to obtain bathymetry with these characteristics by using the methods presented in Herchenroder (in preparation, 1981). The remaining input to the models is a specification of wave height, period, and direction along the models' seaward boundary. Lacking any measurements or first-hand knowledge of deepwater wave conditions at a site, probable values for wave height and period can be estimated from Thompson (1977). Probable wave direction would have to be estimated from other available sources of information; e.g.,

predominant wind direction from the National Weather Service. The wave Information Study, underway at the U.S. Army Engineer Waterways Experiment Station, should also be useful in providing wave climatology at a site.

A limitation of both models is the lack of terms in the equations representing processes known to occur in nature. Both models are propagation models and do not consider effects from wind, other waves, bottom frictional attenuation, wave breaking, or reflection and diffraction. Studies are underway at the Coastal Engineering Research Center (CERC) to include some of these effects in future models.

### III. EXAMPLE OF MODEL APPLICATION

Both models are applied to an area in the vicinity of the CERC Field Research Facility (FRF) at Duck, North Carolina, to illustrate the type of output from each model. Some comparisons are also made to wave measurements taken near the research pier.

Bathymetry in the vicinity of the pier was surveyed in September 1978. The survey data were processed using the techniques described by Herchenroder (in preparation, 1981) to obtain evenly spaced values of depth. Contoured bathymetry for a grid of 50 by 50 lines near the research pier is shown in Figure 1. The cell size was chosen as 82 feet (25 meters); therefore, the grid is 4,018 feet (1,225 meters) on a side.
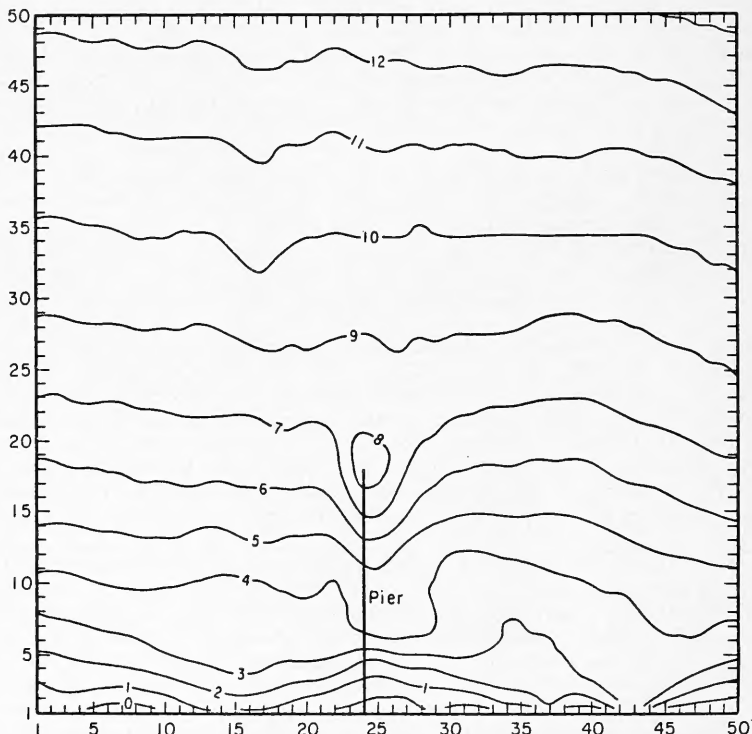


Figure 1. Contoured bathymetry near CERC's pier (September 1978 survey). Contour interval is in meters; grid interval $\Delta x = \Delta y = 25$ meters.

Deepwater wave conditions used as input to the models were determined from Waverider measurements made on 13 September 1978. Input wave conditions for both models are a period of 7.3 seconds, a wave height of 4.2 feet (1.3 meters), and a direction (from which waves are propagating) of 17° counterclockwise from the pier facing seaward.

An example of output from the wave ray program is shown in Figure 2. Information of this type is provided along each ray from its initial to final point. A discussion of these output parameters and other details of the program are contained in the program documentation available at CERC. Results from one of the plotting options of the program are illustrated in Figure 3 by a diagram of wave rays showing the convergence and divergence of rays as they proceed from their initial point offshore toward the shore. Since it is assumed that wave energy is conserved between rays, the energy as the rays converge is confined to smaller areas and results in higher waves. Unrealistically high waves may be predicted in some instances due to the limitations of the theory. No specific criterion is currently available to define such cases; the results must be interpreted carefully in areas of ray convergence. An aid in interpretation is the beta or ray separation factor $\beta$. When this factor approaches zero, calculated wave heights are not valid. How far from zero beta has to be for valid results is a question under investigation. At present, the results obtained in the range $-0.5 < \beta < 0.5$ are questionable.

An example of the output from the spectral model is shown in Figure 4. In comparison to the wave ray model in Figure 3, both models are shown to predict similar patterns of ray convergence and divergence. It is difficult to determine if the ray patterns shown in Figures 3 and 4 accurately depict the actual pattern of waves at the pier for the given angle of approach. A qualitative judgment can be made by comparing a photo of the CERC radar scope (Fig. 5) to the ray diagrams. The continuous white segment across the bottom of the photo is the radar reflection from the shoreline. The line at the center of this segment and perpendicular to it is the pier. The white spot in the center of the photo is the end of the pier where the radar is situated. The other white line segments are reflections from wave crests. A line traced perpendicular to these crests is a wave ray. The pattern of wave crests in Figure 5 shows that there will be a convergence of wave rays to the right of the pier and a divergence to the left. This is also apparent in the ray patterns in Figures 3 and 4 which indicate the model results are qualitatively correct. A comparison of the wave height measured along the pier and that calculated with the two models is shown in Figure 6.

Wave heights are available at each grid point from the spectral model and are plotted from the grid line parallel and closest to the pier. Wave heights from the ray model are available along each ray at points indicated by a (+) in Figure 3. The interpolation program used to obtain evenly spaced bathymetry was also used to obtain wave heights at the grid points of the spectral model using the unevenly spaced wave heights from the ray program. The plotted wave heights for the ray model are thus interpolated values which lie along the same grid line as chosen for the spectral model. Little variation in wave height is indicated from the spectral model. Larger variations are shown for the ray model but that at grid point 25 is suspect since the beta factor is less than 0.5 at that point along the ray and such a difference in significant wave height over one grid distance is unreasonable.

9

FRE WAVE TRANSFORMATION STUDY T=7.3 SEC.,H=4.2 FT.,A=-73.0 DEG.
PERIOD = 7.30 SEC...TIME STEP = 2.19 SEC...RAY SPACING = 1.0...CREST SPACING = 5...BOTTOM APPROX CODE = 1
RAY NO = 1...SET NO = 1

| POINT | X (N.MI.) | Y (N.MI.) | ANGLE (DEG) | DEPTH (FT) | LENGTH (FT) | SPEED (FT/SEC) | REFRACTION COEF | BETA FACTOR | SHOALING COEF | HEIGHT (FT) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | .06 | .69 | -73.00 | 143.53 | 272.87 | 37.38 | 1.000 | 1.000 | 1.000 | 4.20 |
| 5 | .07 | .64 | -77.43 | 37.88 | 217.75 | 29.83 | .999 | .996 | .915 | 3.84 |
| 10 | .08 | .59 | -77.68 | 35.87 | 213.79 | 29.29 | 1.016 | .963 | .916 | 3.91 |
| 15 | .10 | .54 | -77.95 | 35.44 | 208.63 | 28.58 | 1.033 | .931 | .919 | 3.99 |
| 20 | .11 | .49 | -78.23 | 31.37 | 203.89 | 27.93 | 1.051 | .900 | .923 | 4.07 |
| 25 | .12 | .44 | -78.17 | 29.16 | 198.47 | 27.19 | 1.052 | .911 | .928 | 4.10 |
| 30 | .13 | .39 | -77.51 | 27.35 | 193.72 | 26.54 | 1.026 | .961 | .933 | 4.02 |
| 35 | .14 | .34 | -76.80 | 25.05 | 187.22 | 25.65 | 1.002 | .997 | .940 | 3.96 |
| 40 | .15 | .30 | -76.58 | 22.95 | 180.81 | 24.77 | 1.019 | .943 | .949 | 4.06 |
| 45 | .16 | .26 | -77.34 | 21.05 | 174.57 | 23.91 | 1.074 | .848 | .959 | 4.33 |
| 50 | .17 | .22 | -78.35 | 18.93 | 167.00 | 22.88 | 1.136 | .755 | .972 | 4.65 |
| 55 | .17 | .18 | -79.52 | 16.51 | 157.56 | 21.56 | 1.205 | .671 | .992 | 5.02 |
| 60 | .18 | .14 | -81.09 | 14.15 | 147.25 | 20.17 | 1.280 | .597 | 1.017 | 5.47 |
| 65 | .19 | .10 | -83.28 | 11.76 | 135.59 | 18.57 | 1.346 | .542 | 1.050 | 5.93 |
| 70 | .19 | .07 | -85.43 | 9.31 | 121.80 | 16.68 | 1.405 | .499 | 1.097 | 6.47 |
| 74 | .19 | .05 | -87.95 | 7.59 | 110.76 | 15.17 | 1.445 | .473 | 1.143 | 6.93 |

RAY STOPPED, REACHED BOUNDARY

Figure 2. Sample output of the ray program.
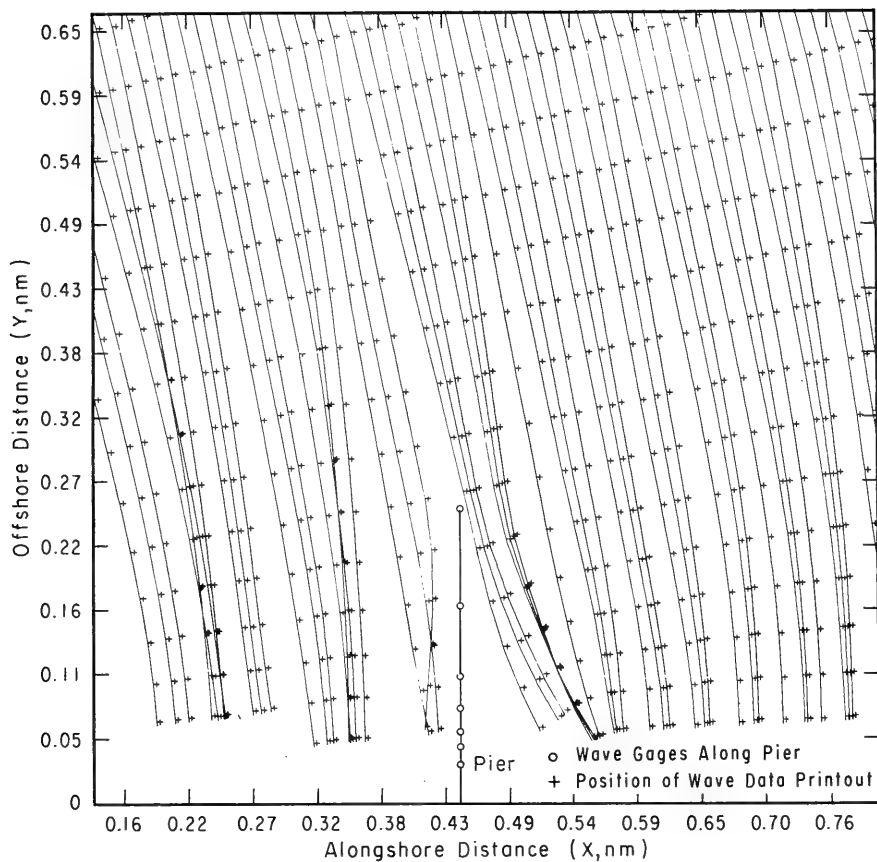
Figure 3.   Sample ray plot showing pier location and
boundaries of grid used in spectral model.
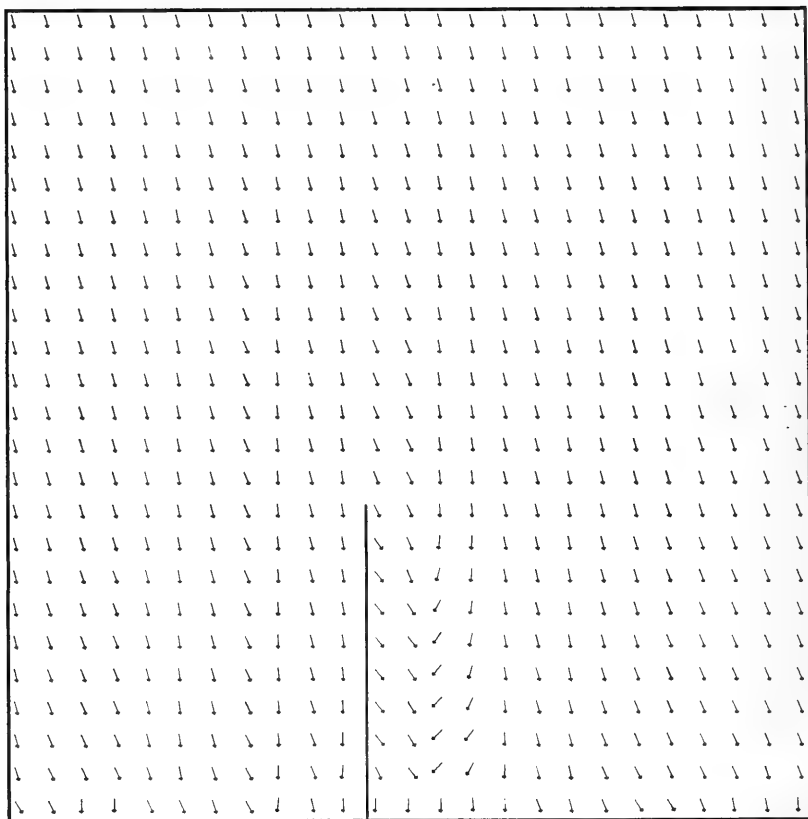
Pier

Figure 4. Plot of wave directions from the spectral model within the region outlined in Figure 3.
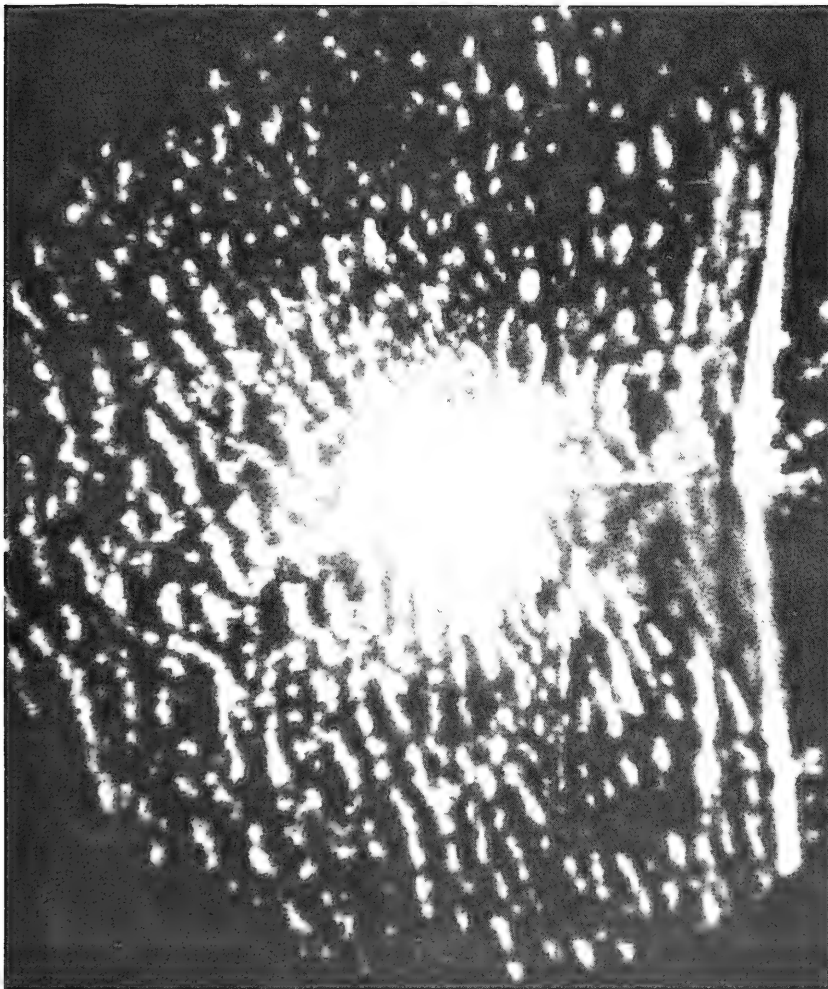
Figure 5. Radar image of wave crests, 2300, 13 September 1978, from CERC's radar at end of pier. Range is 0.325 nautical mile.
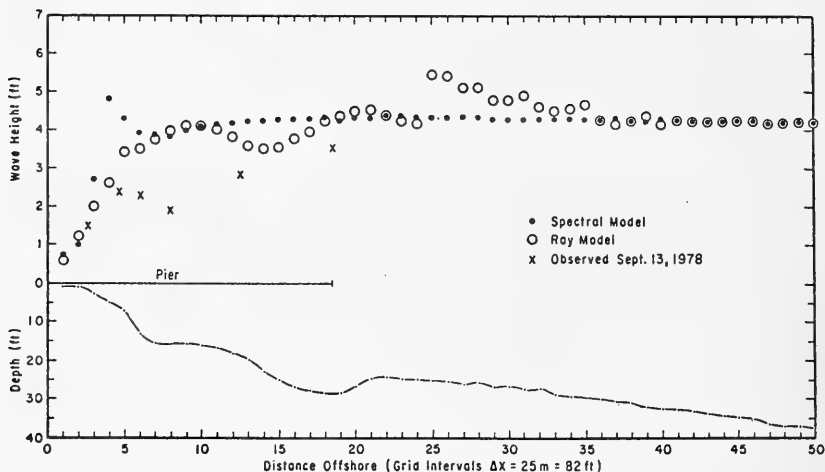
Figure 6.  Comparison of measured and calculated wave heights.

Both model results and observations indicate that there is probably little variation in wave height from the seaward boundary of the models to the region near the end of the pier.  Landward of this point the observations indicate greater reduction of wave height than indicated in the model results.  This attenuation is assumed to be due to frictional and diffractive effects which are not simulated by these models.

Questionable values of wave height and direction can be generated by the spectral model in areas of convergence just as with the ray model.  If a value of wave height or direction differs by what could reasonably be expected over a distance of one grid block (82 feet in this case), it probably is not valid.  As a rough estimate, wave height changes of more than 20 percent in the distance of one wavelength should be considered questionable.  To obtain a more representative value, a neighboring value or average of values can be used.  Work is underway to improve both models in those cases where results are not valid.

IV.  SUMMARY

This report has discussed two numerical models which allow a wave field to be transformed by the process of refraction and shoaling from offshore to nearshore.  One model (WAVE, 720X6R1CH0) gives results along wave rays; the other model (SYLT, 720X6R1CF0) gives results on a horizontal two-dimensional grid of points.  Both models are based on linear wave theory and hence are limited by the assumptions of the theory.

Examples of model output are presented by an application to an area near the CERC Field Research Facility.  Model results and field measurements of wave direction and height are compared.  The ray model (720X6R1CH0) and the finite-difference grid model (720X6R1CF0) may be obtained from the CERC ADP coordinator, Kingman Building, Fort Belvoir, Virginia 22060-5196 or from the Engineering Computer Program Library (ECPL), U.S. Army Engineer Waterways Experiment Station, P.O. Box 631, Vicksburg, Mississippi 39180.  Documentation to assist the user in running the models is also available.

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